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ABSTRACT

This address, presented to the National Council of Supervisors of Mathematics and the National Council of Teachers of Mathematics at their 1978 annual meetings, discusses poor mathematics scores in elementary grades and argues that they are the result of an outdated and outmoded elementary school mathematics curriculum. The origin of the standard mathematics curriculum is outlined and the challenges involved in changing it pointed out. Some of the needs for change are suggested and the general directions they must take indicated. Efforts being made in these directions are described. (MP)

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An Elementary Mathematics Curriculum for ALL Children

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by
Robert Wirtz

A Major Address
Presented to the National Council of Supervisors of Mathematics
and the
National Council of Teachers of Mathematics
at their annual meetings,
San Diego, California
April 1978

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THESIS: Poor math scores and the deeper troubles they manifest are the inevitable result of an outdated and outmoded elementary school math curriculum. Improvement depends on a change in basic policy. We know the general directions changes must take. But such fundamental shifts require the leadership and support of chief school administrators. Until this happens, substantial improvement is extremely unlikely!

I. THE ORIGIN OF THE STANDARD MATH CURRICULUM

The scope and sequence of the standard elementary school math curriculum is the same as it was fifty to seventy-five years ago. Its strategies and tactics have not changed. The only apparent change in materials of instruction is that black and white illustrations have been replaced by colorful pictures. During Century 20, our society has been in transition, growing out of a semi-elitist educational era into a much more vital and democratic school culture. In those earlier years, elementary and secondary schools were assigned the task of weeding out those children who did not prepare themselves to meet the requirements of higher institutions of learning. Pursuant to this goal, the elementary school curriculum was a collection of obstacle courses designed so only those who were well "trained" could reach the finish line. Those who fell down or gave up were channeled into a holding pattern to accommodate non-college bound children.

The math curriculum provided a series of increasingly difficult activities. The responsibility of teachers was to present the material, page by page. The responsibility to "get it" was placed squarely on children; parents were expected to fulfill the role of homework overseers. If children didn't measure up, they were "failed"--retained for another year at the same grade level or shifted into a more "practical" sequence.

Our society evolved over the years, becoming increasingly vital and responsive; education kept pace, setting new and more challenging goals. Creative curriculum leaders and teachers shifted emphasis from "presenting" to "teaching and learning" as an interactive process. School people developed increasing sensitivity to the individual needs of children. They accepted the responsibility of helping all children realize their potentials as fully as possible.

In keeping with these new goals, the math curriculum needed to be revamped to focus on meeting the needs of all children. It needed to turn away from its role of helping to weed out non-college bound children; it needed to be revitalized with new strategies and tactics responsive to many different learning styles.

But this did not happen in very many places. Today's curriculum is essentially the same curriculum that served the purposes of a more elitist

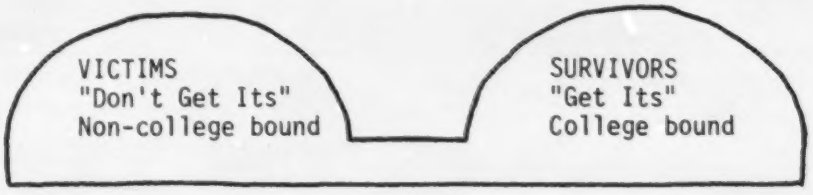
educational system in a different era. It is still a series of bits and pieces of mathematics arranged in an ascending order of difficulty. Most teachers are still expected to "teach" or "cover" the content; children still have the responsibility to keep up, pass more and more tests and not "drop out." The scope and sequence remain intact--the strategies and tactics are unchanged and unrealistic pressures confront teachers and learners alike.

And, of course, the outcomes are the same--the same as they were in the 1920's. By the middle grades, children are effectively separated into two distinct and easily discernible groups. Teachers refer to them as the "I don't get its" and the "I get its." And, as this outmoded curriculum unfolds year by year, the group of victims grows and the group of survivors dwindles.

This tragic polarization, so repugnant to today's society, is masked by the skill of standardized test makers. Test scores show a large population bunched under the peak of a bell-shaped curve with a few stragglers on both sides. This profile has no suggestion of the two widely separated groups that actually exist.

Test makers are asked to create instruments so scores will wrap neatly under the curve of "natural distribution." They use a fool-proof strategy. When the results are tabulated, they retain only those items that about half of the children missed. The rest are excluded. This method guarantees that the results will fit the normal curve, but it provides no useful information about actual conditions. In fact, it hides a disturbing and cruel fact.

If another test were given composed of items rejected by the test designers--items that substantially more than half of the sample population answered either correctly or incorrectly--then the outcome would reflect the bimodal reality that classroom teachers know so well:



The current math program was fashioned many years ago to produce exactly those results! It met the needs of that earlier era....weeding out children who would not qualify for yesterday's institutions of higher learning. It meets today's needs rather less well.

II. A SERIOUS CHALLENGE

There have been few efforts to revise the standard elementary school math curriculum over the years. One major effort was the "new math revolution."

In the late '50's, after Sputnik, mathematicians were asked to look at high school math. They were astonished at what they found and proposed a general overhaul of the content. "New Math" was the result. And it was successful in helping to prepare more than enough research mathematicians to catch up to the Russians.

But overly zealous math educators pushed the new content and abstract language down into the elementary schools--all the way into kindergarten.

What they failed to understand was that our difficulties at the elementary level were not with the "content" in the math curriculum. Rather, we needed to develop more effective strategies and materials sequenced to respect the way children learn and designed to engage teachers in helping children move on toward higher cognitive levels. When viewed as a learning problem first, content decisions become a minor issue.

As "new math" succeeded in bringing about substantial revisions in the elementary school math curriculum, administrators assumed that the only issue was content. Therefore, they called in "experts" to make necessary revisions and to provide in-service training for teachers. Only a part of the problem was being addressed.

The hopes and promises of "new math" failed to materialize. A steady erosion of scores continued until they bottomed out at the dismal level we have lived with for 10 years.

By 1970, most of the proponents of new math--particularly the experts--had turned to other fields of endeavor. Unfortunately, not enough school administrators who had delegated their responsibilities to those "experts" returned to resume their role as leaders. Thus a vacuum was created.

Textbook publishers, out of necessity, moved to fill the vacuum. They did so in the only safe way they knew, returning the elementary math program to its traditional and outmoded format of fifty years ago.

For the past decade, most textbook publishers have gradually re-established the familiar program out of the past with only two slight innovations. They have greatly improved the realism of the illustrations, and they have eased up a little on the content so it would seem to be easier to teach.

III. ADMINISTRATORS LOOK AWAY FROM THE MATH CURRICULUM

Meanwhile, administrators have become engrossed in management issues without paying any critical attention to what they have sought to manage. They seem to have assumed that the traditional program is a "given," and have not recognized the fact that it is actually the crucial variable.

The management infatuation seems to have begun with the notion of "teaching machines." At first "hardware" was popular. This soon gave way to "software" generally referred to as "programmed learning."

This led on to "performance objectives" and "individualization." A popular slogan of the day was "individualized diagnostic/prescriptive approach to learning."

Elaborate "management systems" were created to guarantee "continuous progress" monitored through individual "pupil profiles." Some are beginning to talk about "mastery learning" and the affective domain while others are satisfied to achieve "minimum competencies" to gain exit from schooling.

Tremendous amounts of public funds are made available for organizational efforts. Title I supplies a half-billion dollars ($500,000,000) a year for such efforts to improve the math scores of eligible children. Adult-pupil ratios are dramatically reduced. Shelves are filled with expensive "manipulatives." The number of aides and resource personnel has greatly increased.

All this and more to improve the math scores of "identified" children. But the scores have stayed fixed on the bottom.

There has been periodic textbook switching. There are projects featuring math labs and learning stations and team teaching and ungraded classrooms. Math task forces have been set up at District and State levels. "Exemplary programs" have been identified. Long-term plans have been developed for "restructuring" the schools. But math scores remain stabilized at the same depressed level.

That's a lot of variables--and the list is far from exhaustive! But in every case, the traditional math curriculum has always been taken as a "given," as a "constant." In 1978, only one program challenges the traditional scope, sequence, strategies and tactics. Just a few programs reflect the views of creative teachers and supervisors.

Viewed with this perspective, further reliance on "managing" or "administering" any basic curriculum to achieve meaningful progress in basic skill development is educational bankruptcy--unless the curriculum acknowledges the way children develop and learn and the role that teachers and parents play.

IV. GUIDELINES FOR CHANGE

What then are the curriculum changes that are needed? The following discussion can at best suggest only some of the needs for change and indicate general directions they must take.

Everyone will agree that rote memory and understanding are both essential ingredients of mathematical fluency. If there are disagreements, they are limited to the relative importance of these two factors. It is enough for us to begin by recognizing the importance of both. Any math curriculum must suggest well developed strategies for (1) helping all children remember more in less time, and (2) helping all children understand more mathematics. The standard curriculum is silent on both questions. Or, if not silent, certainly it is fuzzy and not at all specific.

If one were to look for implied strategies in the traditional curriculum, the following might be noted:

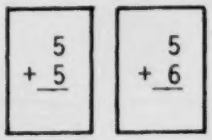
Rote Memory -- "is a matter of sufficient repetitions."

Understanding -- "is telling about an idea often enough so all children will eventually understand it."

Such implications suggest an almost complete lack of understanding of both aspects of learning. They seem to rest on some kind of accumulated wisdom. They are useless as strategies to overcome difficulties children have.

A case history may be helpful:

> Marcia, a fourth grade child, brought home a note asking her parents to work with her with flash cards... "she is still counting on her fingers, and that's not fast enough in 4th grade." (This caught her parents by surprise: they learned that children who are dependent on their fingers can test out "at grade level"--until 4th grade.)
>
> Marcia's father found out quickly what the teachers had reported. When the card "3 + 7 = ____" came up, Marcia used her fingers: "eight, nine, ten...it's ten!" Her father complimented her on counting on from the larger number and for getting the right answer--but with a note of moral judgment, he asked, "Why don't you remember that?"
>
> There was considerable finger counting and more, "Why don't you remember that?"....each time with a slightly higher pitch.
>
> When the card with "5 + 5 = ____" came up, Marcia beamed and said, "Ten," without hesitation. "Ah," her father thought, "an entry point." So he found " 5 + 6 = ____" and put the cards side by side and asked, "Now, how much is 5 + 6?"



Marcia counted on her fingers from six to eleven and said, "It's eleven."

"But Marcia: you know that five + five = ten; six is one more than 5, so if five + five = ten, how much is five + six?"

Marcia was confused and troubled. With brows knit tightly, she counted on her fingers from 6 to 11 and asked, "Isn't it eleven, Daddy!?"

Sparks! "Marcia, don't you see any connection between five + five and five + six?"

"Yes, they both begin with a '5'!"

This first session ended with Marcia knowing she was an "I don't get it," and her father realized he was an "I can't help her."

A. Rote Memory

Why is it Marcia can encounter 3 + 7 = ____ a countless number of times over 4 years and not remember that the answer is "ten"?

Clearly, "sufficient repetitions" is not a productive tactic for many children. A bit of analysis reveals that Marcia has been practicing counting on her fingers . . . and she needs no more practice at that method of finding out. What she has always needed and almost never experienced was saying the complete sentence--"three + seven = ten"--and hearing that sentence from teachers, parents, and other children.

The unit of rote memory is a verbal pattern that, once begun, "rolls off the tongue" as a TV commercial or line of poetry. And this is true whenever rote memory is involved.

You have memorized your telephone number; so you ought to be able to recall automatically the digits indicated below:

XXX-XXX-XXXX

In the Pledge of Allegiance to the Flag of the United States, what word comes just before and what word comes just after the word "Republic?"

How many days in the month of April?

If your last name is long enough, what is the 4th letter in your last name?

Can you complete the sentence below?

"Hey, diddle, diddle, the _____."

No trouble: once begun, the rest rolls off your tongue with no conscious act of mind. It is in your rote memory--as a verbal pattern . . . a unit of memory.

This insight into the nature of rote memory leads to productive strategies and tactics. When Marcia encountered "3 + 7 = ____," she began immediately to read the card--"three + seven = ____," and if the answer didn't "roll off her tongue," she would find out on her fingers that 7 + 3 = 10. "It didn't roll off my tongue that time, Daddy, but I know what to do. I'm going to say the whole sentence 3 times--I'll say it as if I were angry (or happy, or fast or with a beat . . . any way she decided to say it).

When rote memory is involved, classroom teachers and parents ought to police their own language, never settling for answers, but requiring complete sentences--repeating over and over again the full unit of memory. And we might well reinstate to good standing "chanting" or "choral response" . . . which not only promotes memory, but also can embrace the powerful and positive dynamic of group singing. The "I don't get its" and the "I get its" respond together--and the latter group ought not be excluded simply because "they know the words of the song."

B. Understanding

Piaget provides powerful insight into the source of difficulty "I don't get its" experience--and indicates strategies we must develop to overcome those difficulties and prevent them from arising:

> The true cause of failure in formal education is that we begin with language rather than beginning with real and material action.

Language is a repository of experiences. It begins as children respond to the sensory data flooding into their minds as they encounter the three-dimensional world around them--a world of things and people. They see connections and relationships they talk about--to themselves and to others. They use words and language patterns as organizers to draw various aspects of their experiences together. They build mental images and develop language patterns that stay in their minds after the experience passes.

Language grows out of "real and material action"--its meaning depends exclusively on the experiences.

Unfortunately, it is possible to begin with language that has no referent, no meaning. And this is doubly destructive: it develops a language as limited and capricious as the vocabulary of parrots; also, it hides the fact that no meaningful language is being developed--the worthless counterfeit is being accepted at face value.

This is the plight of "I don't get its."

Marcia could not comprehend her father's language that focused on relationships because it had no referent for her. Shortly thereafter her father introduced her to ten-frames--a 2 by 5 array of wire or plastic mesh.

Marcia was asked to put 5 beans in each of two frames (Figure 1).

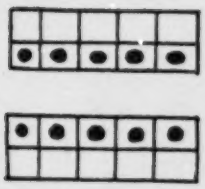


Figure 1

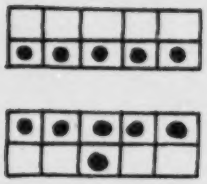


Figure 2

She was asked whether or not she counted the beans as she put them in.

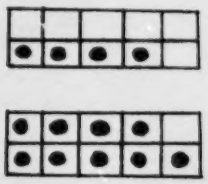
"No, I just filled up one row in each; and I know you want me to say five + five = ten."

"That's right. Now, will you please put another bean in the bottom frame?" She did (Figure 2), and without any hesitation, she said, "Now you want me to say five + six = eleven."

"How did you find that out so easily?"

"Don't you see, Daddy, the 2 fives make ten and one more is eleven?"

A little later she was asked to put four beans in the top frame and nine in the bottom frame. She did.



"Did you count those nine beans as you put them in?"

"No, I just left one hole....and you want me to say four + nine = thirteen. That's easy: you take one of the four and fill up the hole in the bottom frame, and then you have ten and three or thirteen."

She thought a minute and said, "Hey, Daddy, that'll work for all the nines....you can always fill up the hole and have 10 plus one less than you started with in the other frame."

And she proceeded to make up examples without using the frames or beans: "Seven + nine will be six and ten or sixteen"....etc.

Marcia smiled: "From now on, the nines are my 'cinchy' numbers."

She thought for a minute and continued, "And I can use the same trick with the eights--you move over two to fill up the ten-frame: eight plus seven must be the same as 10 and 5 more or 15."

"You know, I think I'll call the eights my 'almost cinchy' numbers."

Not long afterward, she asked, "Daddy, will the nines be 'cinchy' in subtraction?"

She was building a language of connections as it grew out of "real and material action" and now was asking "global questions" searching for broad generalizations.

"I don't get its" are not naturally "language deficient." Rather, they are failing because they were initiated to language that was not rooted in "real and material action."

"I don't get its" are victims of a curriculum that fails to appreciate the central role of language experience and language development in mathematics or fails to help teachers and parents with their responsibility to help all children develop the crucial language of connections and relationships.

"I don't get its" suffer a common difficulty. Each bit of arithmetic is isolated--it stands by itself. The subject is a maze of unrelated items to be remembered without benefit of anything else you know. They never learned the language of connections or relationships. The "I get its" in some way learned that language--they can talk their way from what they know out into less familiar terrain that's close by.

Conclusion

The elementary school math for the '80's must assume the responsibility to help all children memorize what they need to memorize and to help all children understand the relationships that weave all of mathematics into a single tapestry.

It must provide strategies that are responsive to the unique characteristics of rote memory--it must provide strategies designed to build understanding and to develop the language upon which understanding depends.

Finally, it must offer suggestions for weaving the cognitive functions together in all learning activities.

V. AN EXPLICIT AND COMPREHENSIVE THEORY OF LEARNING

The standard elementary math curriculum adheres to no explicit theory of learning. It moves ahead following the dictates of tradition. Its general shape was formed before such seminal thinkers as Jean Piaget and Jerome Bruner had even commenced their research into the ways young children learn. And over its long history, this curriculum has been almost totally unresponsive to the insights provided by Piaget, Bruner, and the other learning theorists. This was perhaps a natural aloofness to be expected during the era in which schools placed the responsibility for achievement on the learners themselves.

But today, schools have set the goal of helping all children learn and of helping to develop the potentials of all children as fully as possible. To meet such a challenge, schools need to respond aggressively to all the insights learning theorists can provide. As we begin to translate theory into practice, we need to move from an eclectic beginning that considered all points of view toward a comprehensive theory of learning that has its own internal consistency.

Children as Learners

All children come to school having demonstrated they can learn almost anything they want to learn. They are experts at learning through "trial and error." They come to school as seasoned inventors. They are full of the aspirations, the hopes, the dreams, and the self-confidence that distinguish human beings from all other animals.

The extension of these natural learning abilities is awesome. At age two or three, children fathomed the basic structure of their mother tongue. When they ask, "Will you read me the same story you readed me yesterday," they are not mimicking--they are inventing a new word according to the standard rules of forming the past tense of regular verbs in English. Early on, they learn to read other peoples' "body language." They develop several reliable ways to control adults.

They have also had many experiences with the four basic operations of arithmetic: subtraction ("take away") and addition ("some more") and multiplication ("so many for everyone") and division with remainders ("fair shares"). However, they are not yet able to verbalize all they know; consequently, the math curriculum must start with providing opportunities for language experiences. As this need begins to be fulfilled, another need arises naturally--to learn more about the "neat" and efficient ways arithmetic provides to record what they have learned to describe with language. But the ability to learn is unquestionable.

Many researchers--notably Piaget--report their observation of this "natural learning process"--the ways children learn who are on their own and not responding to adult intervention. These researchers remain outside,

uninvolved in the activity and, thereby, preserving its "naturalness." They are careful that they do not "contaminate" the process they are observing.

They report that children follow a common developmental pattern, maturing slowly through several stages according to some kind of innate timetable. They describe the basic development of ideas that children finally internalize and learn to use. All such notions are born out of "real and material action"--children responding to the sensory input that floods in from the world about them. As familiarity grows, language begins. Gradually, the three-dimensional context in which the idea originates can be replaced by two-dimensional representations such as pictures, diagrams, sketches and graphs. The language already developed is used to talk about things that can no longer be moved around. Eventually, these representations of the real world also fade out, and the idea is retained as a combination of visual images and language patterns. This slow transition from concrete to abstract is dependent on the continuing development of visualization, language, and thinking.

Other researchers--notably Jerome Bruner--are not content with observing and describing the natural learning process. They want to intervene in that process to help maximize children's development at higher cognitive levels--developing those potentials that are uniquely human. Only human beings can organize what they remember, enabling them to tackle problems and solve them. Moreover, human beings can move on to an even higher level--initiating and carrying out their own independent investigations.

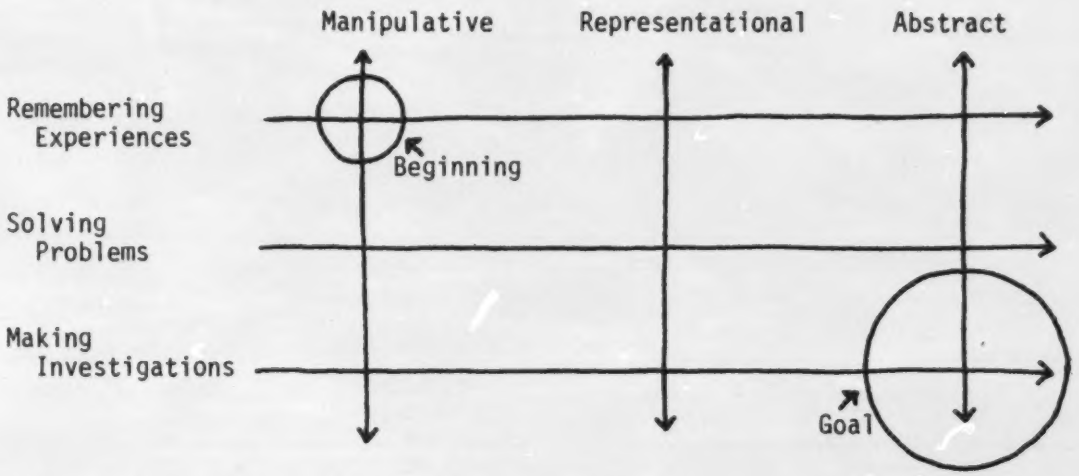
In The Relevance of Education, Bruner has written:

> Discovery teaching generally involves not so much the process of leading students to discover what is 'out there,' but rather, their discovering what is in their own heads. It involves encouraging them to say, 'Let me stop and think about that'; 'Let me use my head;' 'Let me have some vicarious trial and error.' There is a vast amount more in most heads (children's heads included) than we are usually aware of, or that we are willing to try to use. You have got to convince students (or exemplify for them, which is a much better way of putting it) of the fact that there are implicit models in their heads which are useful.

At a single sitting children can move from one cognitive level to another--from remembering experiences, to solving problems, to making independent investigations. Children can move from one level to another and back regardless of their progress along the developmental axis from concrete to abstract. Children can "use their heads."

Any math curriculum that is fit for human consumption must provide a great variety of opportunities for activity at these higher cognitive levels. (This is true even though test makers have been able to monitor progress only at the levels of "remembering experiences.") And mathematics is uniquely rich in problem solving situations that can be tailored to meet a wide range of different needs.

The 2-dimensional diagram below accommodates both the observers such as Piaget and the "intervenors" such as Bruner.



Movement from left to right is meant to suggest a slow developmental process--ideas born in "real and material action" mature through a representational stage and finally grow into useful "abstractions." This is essentially a one-way route. Movement back and forth from top to bottom is meant to suggest that higher levels of appropriate cognitive activities are available to all children at all times. Both movements have their origins in "real and material action."

The ultimate goal of education is shown as helping to provide all children with opportunities to enjoy making "independent investigations" at the "abstract level."

Now, every strategy, every tactic, every activity in the math curriculum can be evaluated in terms of its contribution to continuous development of the process mapped by this diagram.

This theory suggests there are always two optional ways to move ahead--toward increasing levels of abstraction and toward higher cognitive levels.

It also suggests useful strategies to help learners who are confused or frustrated; move back toward a less abstract level or to a lower cognitive level to build a more firm foundation before moving out again.

This explicit theory of learning has been developing over many years. It has challenged me to think about and develop an alternative to the standard curriculum for today's children in elementary schools.

VI. FOCUSING ON THE SIMPLICITY OF ARITHMETIC

The standard elementary school arithmetic program breaks the subject up into a myriad of bits and pieces that few children or teachers or parents ever put back together again. Even for many adults, arithmetic is something of a mystery; some feel they are just not born to be friends with numbers.

Actually, arithmetic is surprisingly simple-minded. If confusion ever arises, it has been induced.

Because we want all children to learn much more arithmetic, we need to emphasize its utter simplicity and reasonableness.

Arithmetic deals with four different situations, each of which has three phases. The phases are:

(1) You start with some objects;

(2) You do something; and

(3) You report the results.

It's that simple and uncluttered.

The four situations, with which all beginners have already had considerable practical experiences, are as follows:

A - Addition:

(1) Start with any given number of objects;

(2) Add a given number of objects; and

(3) Report the total number of objects used.

B - Subtraction:

(1) Start with any given number of objects;

(2) Take some objects away; and

(3) Report how many objects are left.

C - Multiplication:

(1) Start with any given number of objects;

(2) Put that same number of objects in a given number of piles;

(3) Report how many objects were needed.

D - Division:

(1) Start with any given number of objects;

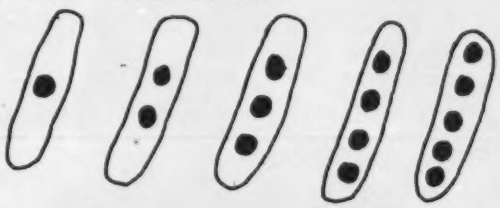
(2) Put them in a given number of piles with the same number in each; and

(3) Report the number of objects in each pile and the number of objects left over if there are any.

That's it!...."real and material action" all the way: four different versions of (1) start with something, (2) do something, and (3) tell what happens. It's all so elementary that most children come to school able to "do" them all. And beyond these simple beginnings, elementary school arithmetic is little more than a never-ending search for ways to avoid the need to count by ones.

But from the outset, these four basic situations or "operations" can be imbedded in challenging "problems"....offering opportunities for children to move to higher cognitive levels--available only to human beings.

Let me illustrate with some examples which will be familiar to those who already know my work. Suppose we have 5 popsicle sticks with 1, 2, 3, 4, and 5 beans glued on them--"bean sticks."



Addition: Can you find a combination of sticks with 10 beans all together? How many different combinations can you find with 10 beans in each?

Subtraction: Can you arrange all five sticks in two piles so there is one more bean in one pile than in the other? Can you arrange all five in two piles so there are 2 more beans in one than in the other?

Multiplication: Can you find a pair of sticks so that one stick has exactly twice as many beans as the other? Can you find another pair with one stick having exactly twice as many beans as the other?

Division: Can you arrange all five sticks in three piles so there is the same number in each pile?

In summary, the basic content of elementary school arithmetic is available to all children:

- counting;
- carrying out four basic activities and reporting outcomes;
- finding strategies to avoid one-by-one counting; and
- experiencing problem solving to develop skills which can be applied to new situations.

All of these activities are available at the outset to all learners as they respond to the three-dimensional world they live in. At each step of the way they can verify everything they say--by applying it to the concrete world around them. They are doing something and reporting what happens. They are developing a language with referents that are part of their experience--language they use to describe the world as they perceive it.

The goal of the elementary school curriculum must be to build on these beginnings, nurturing the development of more visual images and more language rich in solid experiences and growing cautiously, but steadily through increasing levels of abstraction. As this process unfolds, beginning in the manipulation of objects and reaching eventually to the successful manipulation of symbols, language patterns provide the bridge.

The content comes out of the real world experiences of children. Consequently, children can make up or draw or write down situations out of their own real lives of fact or fancy that fit any bit of arithmetic that comes up....the arithmetic is the "headline" and an appropriate "story" fits the "headline." Such opportunities allow children to be themselves, and speak of conditions they understand intimately. They make up their own story problems totally relevant to their own lives. "Applications" that have traditionally been withheld and then introduced as a sort of final test are more appropriate when they are integrated in the on-going learning process.

If the math curriculum were responsive to the needs of children, then no mystery could arise in arithmetic or in its connections with the world children live in.

VII. SCOPE AND SEQUENCE

In the standard curriculum, "carrying and borrowing" or "regrouping" is postponed until the middle of second grade....that is, after the first half of the primary program. Up until then, there have been dozens of pages of drill exercises, but no number larger than nine can arise in any column. Children are asked to complete hundreds of examples involving "two digit" and "three digit" numbers:

27	85	570	876
+ 62	- 63	+ 419	- 371

Bu children see each example as 2 or 3 basic facts. They are developing t.. habit of writing below the line the sum or difference of the two numbers in each column. As they think and compute, no number larger than nine can arise. This encourages the kinds of errors that continue to arise later.

19	148
+ 3	- 79
112	131

In the latter half of the second grade, "carrying and borrowing" or "regrouping" or "renaming" is introduced with many pages in which all examples call for this new skill. At this interval, a child was given the following examples:

1	1
38	32
+ 26	+ 26
64	68
Figure 1	Figure 2

Her mother asked her to "please work out loud."

Figure 1: "8 + 6 = 14....put down the 4 and put 1 in my pencil. Then I put 1 up here."

"Is that the 1 you put in your pencil?" her mother asked.

"No, my pencil is full of 1's."

Her mother was puzzled. "Why do you put that 1 up there?"

"To get the right answer."

Figure 2: "2 + 6 = 8," she began.

"Aren't you going to put 1 in your pencil?"

"No, I don't have any 1 to put in. Next you put 1 up here," she said, writing 1 in the tens-column.

Her mother asked again, "Why do you do that?"

The daughter's somewhat impatient reply was, "I told you--it's to get the right answer"....and she finished the example "32 + 26 = 68."

Many recurring errors such as those given above are the price many children pay for a scope and sequence that unnecessarily develops bad habits....they learn to follow rules without reason and don't always remember the special conditions under which a given rule is appropriate.

The standard scope and sequence postpones "multiplication and division" until the third grade. Actually, all children come to school with a highly developed sense of "sharing" and handling what's "left over." They are doing "division with remainders" long before they reach kindergarten. Many playground fights grow out of disputes about the fine points of "sharing" or "dividing."

The standard scope and sequence ignores or rejects children's demonstrated capacity to learn....children come to school as credentialed learners particularly adept at learning through trial and error. This powerful technique is promptly confronted with its opposite: the directive to "remember right answers." Later on, we ought to understand why it is so difficult for children to deal with the tremendously useful notion of "estimation" which asks for something less than a "right answer." We find children are increasingly reluctant to question whether their results are "reasonable"--another form of estimation.

Richard Feynman, Nobel Laureate in physics, complained bitterly about the fact that the elementary school math curriculum discourages "trial and error." He noted that in his work on the frontiers of theoretical physics, "trial and error" was his most indispensable tool or method. The "right answers" syndrome of the math curriculum for beginners destroys their basic confidence in the "trial and error" they have learned to use so effectively before coming to school.

The standard scope and sequence makes a mockery of the natural or developmental learning sequences Piaget has so insightfully described. With deep-seated wisdom, primary grade teachers complain about this condition in their almost unanimous refrain, "they make arithmetic become too abstract too soon." The curriculum has at best a passing concern for "real and material action" and hurries on to lots of pencil and paper arithmetic.

It is quite obvious to anyone who has a critical eye that the standard "scope and sequence" will continue to discourage and defeat a very large number of timid, eager, unsure, creative, and insecure learners.

For the great population of "I don't get its," the traditional scope and sequence is an unfriendly obstacle course.

VIII. RELATEDNESS VS. RANDOMNESS

There seems to be an unwritten rule that "drill and practice" examples need to be "randomized." Perhaps the rationale is that computational needs arise accidentally and without any connection with similar or related examples. This is, of course, true. For this reason, tests of computational facility need to be randomized.

But "drill and practice" must be evaluated as "learning experiences," not as "testing situations."

Standard materials of instruction would not put these three examples side by side

$$\begin{array}{rrr} 5 & 5 & 5 \\ +\ 5 & +\ 6 & +\ 7 \end{array}$$

because learners might "cheat": they know that 5 + 5 = 10, and that might help them with 5 + 6 = ____. . . children might use what they know to reach out to deal with what they don't know! That's cheating?!

$$\begin{array}{rrrrr} 7 & 7 & 7 & 14 & 14 \\ \times\ 2 & \times\ 4 & \times\ 8 & \times\ 8 & \times\ 16 \\ 14 & 28 & 56 & 112 & . \ . \ . \ ? \end{array}$$

The relationship above might be called the "doubling principle." When one of the factors in multiplication is doubled, then the product is doubled. This sequence shows this relationship among three basic facts and then extends beyond those facts, enabling one to know at a glance that 16 x 14 = 224 and that 16 x 28 = 448, . . . etc.

Relatedness is a characteristic of useful drill and practice. Randomness is appropriate only in the testing mode. When drill examples are related, there is something to talk about.

Consider the following sequence:

$$\begin{array}{rrrrr} \text{a.} & \text{b.} & \text{c.} & \text{d.} & \text{e.} \\ 44 & 45 & 50 & 54 & 55 \\ +\ 44 & +\ 45 & +\ 50 & +\ 54 & +\ 55 \end{array}$$

What do you notice?

- "only three basic facts arise: 4 + 4 = 8; 5 + 5 = 10; and 0 + 0 = 0."
- "if 44 + 44 = 88, then 45 + 45 must be 2 more than 88--or 90."
- "if 50 + 50 = 100, then 54 + 54 must be 8 more--or 108 . . . and 55 + 55 must be 2 more than 108--or 110."

- "only examples b and e present any need for carrying or regrouping, though c, d, and e require using the hundreds column in writing the sum."

Such discussions add another dimension in developing computational skills . . . reaching beyond carrying out simple calculations according to established procedures. In this way, learners develop self-confidence and become more friendly with numbers.

IX. "BACK TO THE BASICS"

By now it should be clear that I believe it is the scope and sequence, the tactics and strategies of the standard math curriculum inherited from a long-gone era that are the obstacles that block every effort to make any significant progress. I also believe that those who fail to understand this are apt to be drawn into nonproductive discussions of content.

Consider the position statement of the NCSM on "Basic Skills":

> "The currently popular slogan 'Back to the Basics' has become a rallying cry of many who perceive a need for certain changes in education. The result is a trend that has gained considerable momentum and has initiated demands for programs and evaluations that emphasize narrowly defined skills."

The statement decries these now narrow definitions and goes on to identify ten vital skill areas, only one of which is computational skill as traditionally defined.

It is tempting to applaud this statement as far as it goes, for I agree with every one of the skill areas listed. However, I fear that any attempt to redefine "Basic Skills" will lead to a nonproductive clash of value judgments. We ought to escape from that trap by asking "Back to Basic" advocates to outline the strategies and tactics they suggest we use.

Typically, they have given very little thought to programs that might be more effective in helping children "memorize" and "understand" the Basics. They will drag up the strategies out of the past--"teach the basics and fail the children who don't learn"--the slogan of an elitist society. Many will call for "more drill and practice" probably unaware that more of the "drill and practice" of the standard curriculum variety will produce more "I don't get its" faster.

The question is not <u>what</u> is Basic, or <u>whether</u> we should focus more attention on basics, but <u>how</u> can we do that in ways that help <u>all</u> children become fluent in arithmetic computation.

For many years now, we have used billions of tax dollars and numerous administrative strategies that could not rock scores off bottom. And unless we are willing to challenge the standard math curriculum, we must accept the fact that we cannot improve general arithmetic fluency. The "Back to the Basics" advocates deserve that kind of straightforward answer.

Unfortunately, the NCSM Position Statement does not place explicit responsibility where it belongs--on the outmoded strategies and tactics of the standard curriculum.

Rather, the Statement debates which skills should be taught--without a word about the strategies and tactics that have some chance of moving us steadily toward our goal. Moreover, as NCSM lists the "Ten Basic Skill Areas" it becomes apparent that they are not describing the standard curriculum at all. We find ourselves wishing two wishes: (1) we wish the standard curriculum was centrally concerned with those basic skill areas, and (2) we wish we had prepared ourselves over a long period of time to help all children develop those skills.

Consider the first skill listed:

Problem Solving

> "Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations...."

If that exciting statement was an accurate reflection of our elementary school math curriculum, then there would be no need for this discussion; scores would go up and all children would develop more low level as well as high level skills.

NCSM might serve our interests far better by first painting that exciting dream, and then evaluating the standard curriculum as it deals with "problem solving."

One of the most serious indictments of the math curriculum is that it has been squeezed almost bone-dry of all "problem solving." In fact, it pays less attention today to problem solving than it did in the 1920's.

Make a search through any current math textbook series for any semblance of a "problem"--one that would qualify under NCSM's definition. When I made such a search, there was none in the first grade book. But near the end of the second grade book, there was a very unusual example:

> "Name four ways to make change for a quarter."

This direction was notably different: (1) it was the first example that had more than one number as an answer, and (2) it was the first example that permitted any choices. (There are more than four ways.)

Comments in the accompanying Teachers Guide are revealing. There was a red flag warning the teachers to "beware" since "children will come up with different answers." There were no more occasions in which such warnings were needed.

Actually, that exceptional example is a teaser: if there are more than four ways to make change for a quarter, are there as many as ten ways? How would you go about finding out how many ways there are in all?

Now we have a "problem" that meets the description of the NCSM statement:

> "Problem solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error."

NCSM deserves vigorous applause for describing "problem solving" as the principal reason for studying mathematics, but NCSM also has a professional responsibility to reveal the fact that the standard elementary math curriculum avoids problem solving and is concerned almost exclusively with nothing more than low level, computational skills. In fact, that standard curriculum is seriously deficient in helping children develop any one or all of the "ten basic skills" NCSM identifies. Evidence grows that high level cognitive skills and computational skills can be developed together for all children.

"Back to the Basics" advocates are sure to be singularly unimpressed with the NCSM list. They will interpret such descriptions as avoidance of the problem they keep raising--math scores are down.

Nor will they be persuaded by the argument that "the tests measure only a sample of the content that makes up the program: certain outcomes are not measured at all." They will want to know why the scores on the "sample" tested below expectations? And why don't we raise them?

There is only one honest answer: we cannot raise those scores substantially as long as we are saddled with a basic curriculum designed to "weed out" about half the children. And we cannot alter that situation substantially unless mathematics supervisors, with teachers and principals, convince district and chief school administrators to initiate and support basic policy changes.

Perhaps if we can be this honest, "Back to the Basics" advocates might join with us in calling for revision of the curriculum so it would enable us to move on toward today's goals of helping all children learn more arithmetic, and helping them develop higher level cognitive skills as well.

X. LOOKING AHEAD

Over the years a growing number of individual teachers, clusters of teachers, principals, supervisors, and a few superintendents and their supporters have pushed out beyond the standard math curriculum. Some have been bold enough to trade off the standard curriculum. Most have been lonely, but active. They have pledged their allegiance to children and dedicated their efforts toward helping all children learn more. When they realized that the "accepted" curriculum was an insurmountable object, they tried to fashion parts of their own curriculum and kept searching for a basic curriculum which was flexible enough to respond to ways children learn.

These pioneers are ready and anxious to welcome newcomers to the frontiers--and they are willing and able to support vigorously an innovative policy that respects all children as competent learners. I count myself among the pioneers, and the CDA Math curriculum which evolved from our efforts is an alternative to the standard program.

A Documented Breakthrough

Using the CDA Math curriculum, results of "breakthrough" dimensions were achieved in two carefully documented ESEA Title III projects in the Berkeley Unified School District (CA). The first included 1,000 primary grade children; the other about 1,000 intermediate grade children. Standardized tests showed about 1.5 months gain for each 1 month in school. The bottom two quartiles were substantially depopulated.

There were two variables in both projects--(1) basic materials designed to be responsive to the needs of all children (CDA Math) and (2) a program of professional growth focusing on implementing those materials and creating appropriate supportive materials.

The California State Department of Education selected an independent agency to evaluate the project. Year-by-year summary reports on these projects are available through the State Office, 721 Capitol Mall, Sacramento, California 95814 (Project 1064: Math in the Real World and Project 1668: Computational Fluency Through Problem Solving).

This hard data supports the assertion that "math scores can be raised" when the standard curriculum is replaced by strategies and materials designed to help all children learn.

Many districts, schools and classrooms have undertaken similar projects with similar results. While they are not as well documented by outside evaluators, test scores, pupil attitudes, and teacher attitudes consistently surpass expectations. Without exception, they report a most dramatic change in the attitudes of those children who had gravitated into the "I don't get it" group....they have hope at last!

Impact Projects

During 1977 - 1978, three substantial "impact projects" are being launched in Atlanta, Georgia; Chicago, Illinois; and Detroit, Michigan. Smaller cities like Montebello and Novato in California have been joined this year by Jordan, Utah. All have solid administrative support and well-designed evaluation components.

As in the Berkeley projects, three of the variables are (1) basic materials responsive to children's needs (CDA Math), (2) on-going programs of professional growth focusing on implementation of the CDA basic curriculum, and (3) extending the basic curriculum to respond to local variations and needs.

A fourth factor for change has been introduced in all of these projects--offering training and support for school principals and their math resource people to assume the initiative and to provide on-going leadership for the professional growth of their teachers. A national faculty of professionals has been emerging who have been helping other teachers and administrators to revitalize elementary mathematics.

These "impact projects" also include development of planned programs of extensive parent involvement. Special strategies have been developed and special materials have been created to help parents become actively engaged in the instructional process using effectively the unique opportunities they have for one-to-one time with children.

Top administrators who want to begin developing and implementing a math curriculum consistent with today's goals can become part of this determined and dedicated group. They can be part of a movement in which the Big Cities are showing leadership in breaking through one of the awesome educational problems that has accumulated as we move on into an increasingly vital and democratic society. These cities are unique in another significant way; mathematics supervisors were key to influencing the leadership staff and in encouraging teachers to try an alternative curriculum as a place to begin again or a model to go beyond.

CONCLUSION

This was not intended to be a detached analysis of a sticky academic problem.

Rather, it is an emotional appeal in behalf of so many children weeded out by "the system" who must suffer the stigma of membership in the "I Don't Get It" group.

It is an emotional appeal in behalf of those many classroom teachers who find no satisfaction in teaching arithmetic because giving all they have isn't enough for so many children.

It is an emotional appeal in behalf of those courageous pioneers who dared to step outside the curriculum guidelines so they could respect children's individual needs.

It is an appeal to administrators--particularly chief school administrators--who are dedicated to helping schools meet the needs of all children in a democratic society: please resume your leadership by initiating and supporting ongoing curriculum development in elementary school mathematics and programs to provide the appropriate professional growth necessary to guarantee successful implementation.

The next Revolution in elementary school math will be led by school administrators and teachers together who begin with the conviction that all children can learn much more mathematics than they are learning.

CDA MATH is the program Robert Wirtz has developed as an alternative to the standard curriculum for today's children in elementary schools.

It consists of:

PUPIL MATERIALS

-- Patterns and Problems, Levels A-F

-- Individualized Computation, Levels a_1-f_2

-- My Progress Book, Levels abc, def

TEACHER MATERIALS

-- Drill and Practice at the Problem Solving Level, volumes 1 and 2

-- Banking on Problem Solving, volumes 1 and 2

-- Mathematics for Everyone

PARENT MATERIALS

-- Making Friends with Numbers, addition and subtraction

-- Making Friends with Numbers, multiplication and division